Introduction

Doraemon is a manga series and a name of a walking cat-shaped robot that came from the future to change youth protagonist's life using future gadgets that act in magical ways. It is a nationwide known character in Japan and has been airing in Japan for more than 40 years.

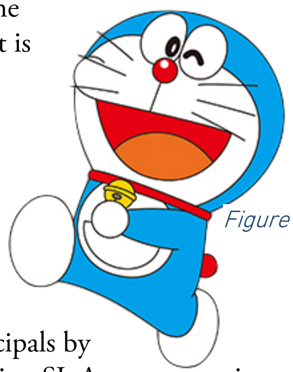


Figure 1

As a citizen of Japan, I, too became fond of this series since when I was small. Although now I understand that the technology probably would not be advanced while I am alive, as a child I genuinely believed that his gadgets are possible sometime soon. It was around that time that I started developing interests in industrial design— which start off as a mere sketch of imaginary items that I wanted to make. Doraemon was a huge source of inspiration in those ideas, and eventually led me in studying design principals by myself. This interest is also reflected in my MYP Personal Project 'Typography' and taking SL Art in Diploma Programme.

Since I aim to pursue a career in industrial design, I would take this math investigation as an opportunity to use my skills in *Adobe Illustrator,* — a graphics editing software commonly used in an industry — that I have developed throughout the years. In scope of finding the aim, this would support further development of my career as an industrial designer by demonstrating my spatial perception ability as well as designing a unique solution to the proposed issue (in this case finding the volume).

Aim

I would be able to utilise my skills in industrial design to simply steps, and to develop modeling skills by first, creating *Doraemon*'s model graphically, then applying skills in mathematics of transformation and calculus (volumes of revolution) to find out his volume from limited information I am provided of – which is a skill that is required for a professional designer.
By finding out the volume, it would allow to apply this skill when full-scale modeling, which is again an important skill in assessing the ergonomics of a product. Silicon casting is typically a popular option in molding. The found volume would specifically be applicable in knowing the volume of silicon to be used in this process.

Splitting up into sections

Before I dive in to calculations, in order to make this investigation intuitive, I have used a 3D Doraemon figure I had since I was small. The 3D model I own is vertically symmetrical, which is significant for this investigation as when using volume of revolution method, it will revolve the shape 2π radians, thus requiring the shape to be symmetrical around the axis of symmetry.

Since Doraemon has a complex shape, it must be split into different sections. Three main section that comprises his body are: Head (Face), Body, and feet. Exterior elements such as nose, arms, pocket that stick out of the main body parts will be calculated individually and added on to the original volume, as these elements cannot be integrated to the original equation.

Figure 2 – Doraemon figure calibrated to 10cm height

To start off, the picture of Doraemon was calibrated on *Illustrator* so that it would have a height of 10cm. In this process, a picture of Doraemon was taken straight above, and the picture was scaled. The scaling is performed in order to provide ease when magnified to the actual scale the creator of Doraemon specifies.

Defining the height would also signify that the whole of body elements across the *x*-axis would add up to 10cm. Since I would be using multiple equations to recreate his body, this would support in specifying the domain as well.

Basic Equations

Based on the adjusted height, I have created an over view of Doraemon's body using *Illustrator*'s Ellipse tool and Pen tool to visualize the outline of his shape. Ellipse tool can create shapes based of circle, and
Pen tool can draw any sort of lines. In both cases, the software will provide the dimensions (width and height) of created shapes.

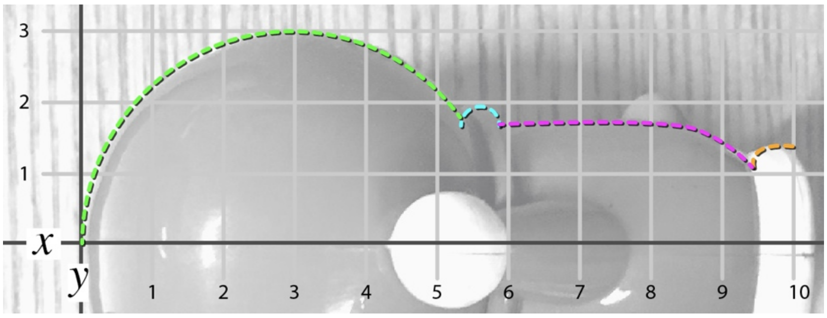


Figure 3 – A graphical representation of Doraemon using Illustrator

Green line is used for the head.
Blue line is used for the bump on the neck.
Pink line is outlining his stomach and leg area.
Orange line is used for his legs.

From this process, I was able to identify two main types of equation that can be used in order to recreate his body structure. Since green and blue line were created using *Ellipse* tool and adjusted accordingly, we can here apply transformation to the general formula of circle, where r is the radius:

$$x^2 + y^2 = r^2$$

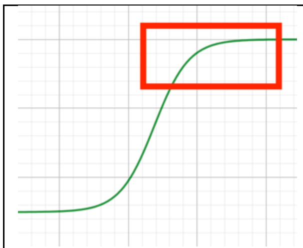


Figure 4 — an example of Logistic function

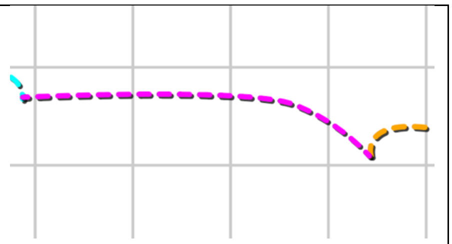


Figure 5 — the close up of orange and pink line

On the other hand, the shape of pink and orange line resembles the shape of a logistic function (the red box area of Figure 4). Although the shape of pink line is opposite to that of the logistic function, this can be solved through transformations. For the orange line, no problem is seen.

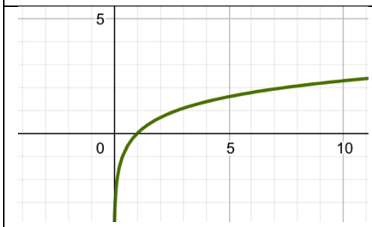


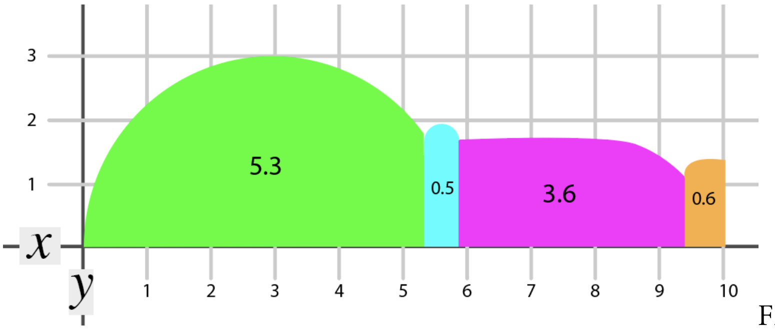
Figure 6 — ln(x) function

It may seem that orange and pink line also resemble $\ln(x)$ function too. However as soon as I graphed, I realized that it is not suitable for modeling, because y value kept increasing as x increased, while as seen in Figure 5 the lines are rather straight after a while.
Overall, logistic function be used for its nature of being able to set limits for plateau. The general formula is below.

$$y = \frac{C}{1 + A \times e^{-Bx}}$$

Where C is the horizontal asymptote, A and B being the constant.

Domain



The values on Figure 7 are numbers showing the width of each element. This would be used to determine the domain of a function. Table 1 is a summary of what domain each function will be using.

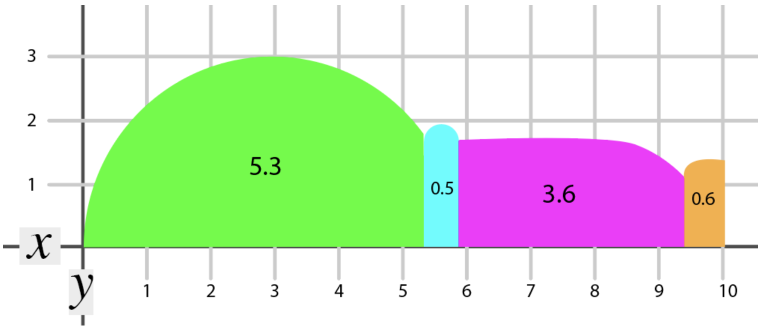


Figure 7 — Tracing of a Doraemon with width of each element indicated

Table 1

Part	Domain
Head	$0 \leq x < 5.303$
Neck	$5.303 \leq x < 5.840$
Body	$5.840 \leq x < 9.384$
Feet	$9.384 \leq x < 10$

alculations of main parts

Head

As stated above, head will use the circle formula. To begin with, following basic transformations must be identified.

$$(x - h_1)^2 + (y - k_1)^2 = r^2$$

While both h and k indicates the coordinate of the circle's center, h translates the x-coordinate, and k translates the y-coordinate. In this case, we can see that the green line directly extends from the origin (0,0) (Figure 3/) and does not require a vertical translation.

$$k_1 = 0$$

The dimensions of this green line are also provided by the software that was used to create this model. Since the graphical representation was created by adjusting the circle, semi-circle, the maximum height (*y-axis)* of this line would provide us the radius.

$$r_1 = 2.968 \text{ (given)}$$

The value of $\boldsymbol{h_1}$ **is also same as radius**, as we want the circumference of the circle to lie on (0,0), not anywhere negative.

Overall, following is the equation of the head after substituting the values back in.

$$(x - 2.968)^2 + y^2 = 2.968^2$$

Rearranging:

$$\begin{aligned} y &= \sqrt{2.968^2 - (x - 2.968)^2} \\ &= \sqrt{8.809024 - (x^2 - 5.936x + 8.809024)} \\ &= \sqrt{5.936x - x^2} \end{aligned}$$

As previously mentioned, the software provides dimensions for the shape created. From this data, I am also come to aware that the ellipse is not a complete circle, and it is transformed slightly to match the outline. According to the software, width of the circle is 5.764cm, and height is 2.968cm.

Dividing the width by 2 will give enough information to find out the scale factor.

$$\frac{5.764\text{ (width)}}{2} = 2.882$$

I now can adjust the width of the equation to 2.882 cm by applying horizontal compression.

$$\text{scale factor of } x = \frac{2.882}{2.968} = 0.9710242588$$

Applying the transformation:

$$y = \sqrt{5.936\left(\frac{x}{0.9710242588}\right) - \left(\frac{x}{0.9710242588}\right)^2} \quad \{0 \le x < 5.303\}$$

From the calculation above, function for the head $f(x)$ is found.

$$f(x) = \sqrt{6.17x - 1.08x^2} \quad \{0 \le x < 5.303\}$$

Neck

The bump of his neck, in context is a result of him wearing a choker with a bell. Since it is a choker made of strings, it is fair to assume that the cross section is a perfect circle. Therefore, it does not require enlargement or compression.

Letting $g(x)$ be the function of his neck, we know that:

$$g(x) = \sqrt{(r_2)^2 - (x - h_2)^2} + k_2$$

According to the information the software provides, the value for radius is following:

$$r_2 = 0.27$$

In order to allow multiple equations to seamlessly connect with each other, y-value at the domain limit of Head equation was found and applied as a transformation so that two equations will not overlap.

$$f(5.303) \approx 1.610$$

$$Vertical\ translation\ by \begin{pmatrix} 0 \\ 1.610 \end{pmatrix}$$

$$k_2 = 1.610$$

The centre of the circle was also moved for the same reason:

$$Horizontal\ translation\ by \begin{pmatrix} 0.27 + 5.303 \\ 0 \end{pmatrix} = \begin{pmatrix} 5.5703 \\ 0 \end{pmatrix}$$

$$h_2 = 5.5703$$

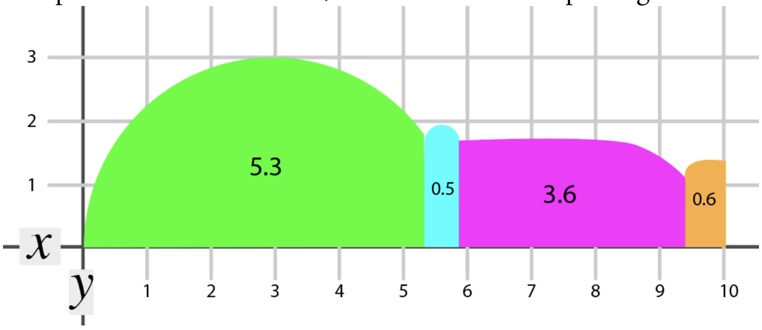
Final equation of the neck:

$$g(x) = \sqrt{0.27^2 - (x - 5.5703)^2} + 1.610 \ \{5.303 \le x < 5.840\}$$

Body

While logistic function exists, it is much complicated than the circle formula to transform and match the shape. Thus, manually adjusting the formula is both time-consuming, and human-eye is never accurate to begin with.

I sought for a solution where I can use the existing <u>known data</u> that could be reused mathematically in a form of equation. Initially, free mathematics software *GeoGebra* was used to plot data points on a photograph, but I faced a problem where I was not able to calibrate the plot scale as I desired to, which resulted in exporting a dataset that was nowhere close to the scale seen in Figure 7.



As I proceed, I found a free software *'WebPlotDigitizer'*, on the internet. This software was developed aiming to 'reverse engineer images of data visualizations to extract the underlying numerical data' (Rohatagi A., 2018), which suits perfect for my situation where I have a visual representation without numerical data, but with existing scale.

After exporting a higher resolution image from *Illustrator* and imported into *WebPlotDigitizer*, I was able to successfully calibrate so that the scale of *x* and *y* would match up to that of the image. Following this, I have plotted points on the image along the pink line to uncover the numerical value of the body outline. The plots were then extracted for further analysis.

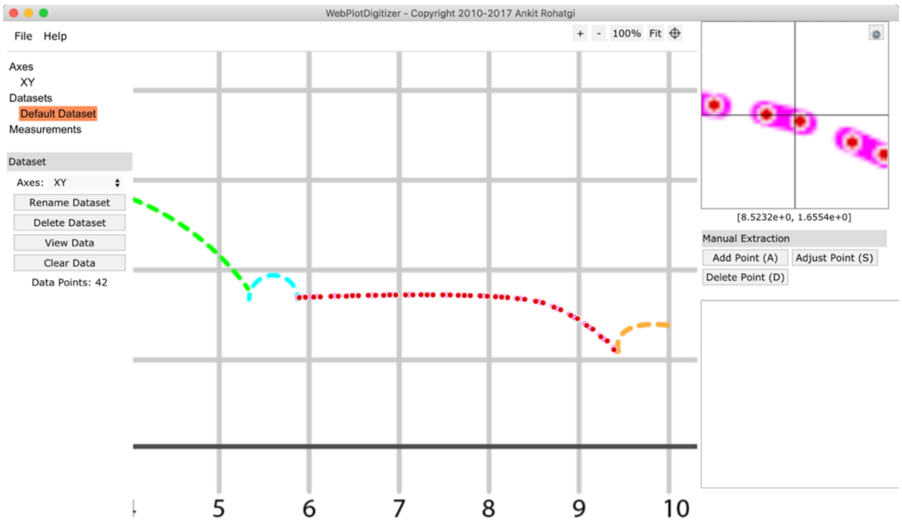


Figure 8 – Process of plotting red dots on the Pink line. Check Appendix A for raw data.

Datasets extracted were analysed in *GeoGebra*. This software will output the equation for the binary logistic regression from datasets using *=FitLogistics(Dataset)* command.
A regression is a statistical method of summarising the finding between independent variables and dependent variables. Although this case, there no such thing as variables. Despite this, the core aim for the regression is to minimize the residual error (the difference between actual value and predicted value) when compared with the actual data sets, and *GeoGebra* will aid in minimizing this.

1st attempt

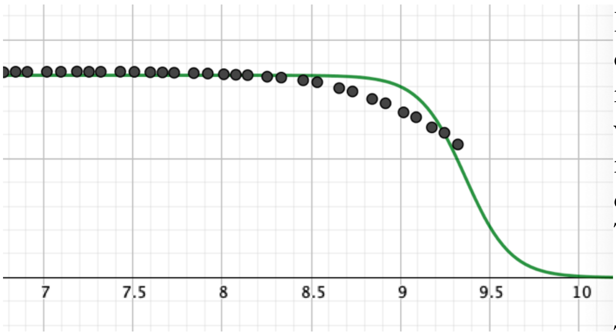


Figure 9 – Initial attempt on Geogebra

In Figure 9, the black dots are the extracted data and orange line is the best-fit *GeoGebra* gave. As it is apparent from Figure 9, the best-fit command did not work very well. This is perhaps because as seen in Figure 4, a logistic function show an exponential growth rather than exponential decay.
The equation exported was following:

$$y = \frac{1.69957}{1 + 0e^{7.811x}}$$

The accuracy of this model was measured using the R-squared (R^2) value.

R^2 is a statistical measure of how well the trend is being represented in a model, from scale of 0 to 1, where 0 being the least accurate and 1 being a perfect model.
R^2 for Figure 9 was **0.786**. The high R^2 value despite the discrepancy between real data perhaps arises from the data plots that are plotted horizontally.
The R^2 for the plots located in between $8.5 < x < 9.5$ in particular was **0.396**, further indicating that the model generated here is inaccurate.

2nd attempt

I assumed that this error could be because the shape Figure 5 had was mirrored compared to normal shape of logistic function (Figure 4). Therefore, I performed a horizontal reflection for all the datasets extracted, and thus would match the shape seen in logistic function.
Additionally, the whole data set was moved to quadrant 1, as few of the plots after reflected invaded quadrant 4. Following translation was applied:

$$Translation\ by\ \binom{9.322465}{0}$$

-9.322… was the smallest value amongst the data plots, and translation had to be greater than this to make everything in quadrant 1. This point now lies at $x = 0$.

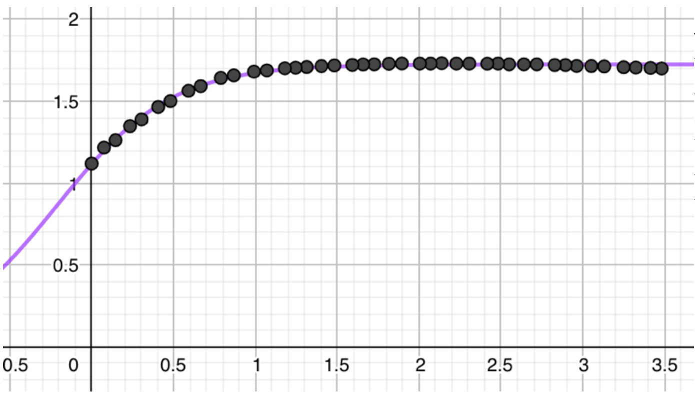


Figure 10

After reflection and translation, *GeoGebra* was able to give out an equation of a logistic model that fits the dataset.
R^2 value for this refined model was 0.995, in which is a fair value to conclude that it is accurate enough to be used.

Following is the equation generated by *GeoGebra*

$$i(x) = \frac{1.72496}{1 + 0e^{-2.85501x}}$$

As I applied several transformations to obtain this equation, few additional steps must be taken to make this model usable. First, the process of horizontal reflection must be reversed.

$$\text{Horizontal Reflection: } i(-x) = \frac{1.72496}{1 + 0.54605e^{-2.85501(-x)}}$$

$$\text{Translation by } \begin{pmatrix} 3.544 + 5.84 \\ 0 \end{pmatrix} = \begin{pmatrix} 9.384 \\ 0 \end{pmatrix}$$

$$i(-(x-9.384)) = \frac{1.72496}{1 + 0.54605e^{-2.85501(-(x-9.384))}}$$

Therefore, the final function $h(x)$ for his body is $i(9.384 - x)$:

$$h(x) = \frac{1.72496}{1 + 0.54605e^{2.85501x - 26.79141384}} \quad \{5.840 \leq x < 9.384\}$$

Feet

Similar procedure as *Body* section was used as well for the feet. *WebPlotDigitzer* allowed me to access the numerical data points of orange line, and the datasets extracted from this software was imported to Geogebra, which gave the following plots and model:

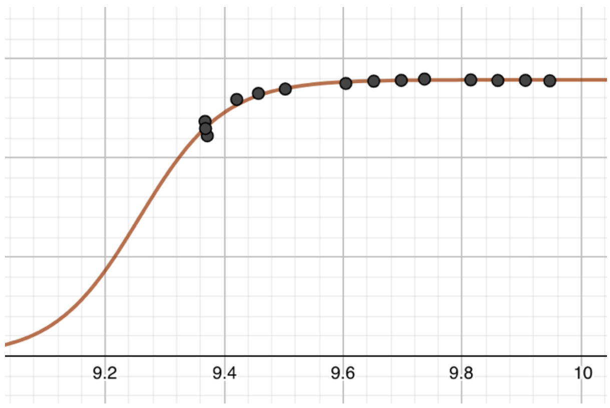


Figure 11

The R^2 value for this function was 0.965, in which is a fair value to say its accurate.

No transformations are required for this model, as I did not perform any of them, as plot resembled the shape of a logsitic function from the beginning.

Final equation:

$$k(x) = \frac{1.39413}{1 + (28255299675732400588 \times 10^{37})e^{-14.04145x}} \quad \{9.384 \leq x \leq 10\}$$

Main components on graph

Now that I am done finding 4 main equations that comprise the body, I have graphed and achieved following:

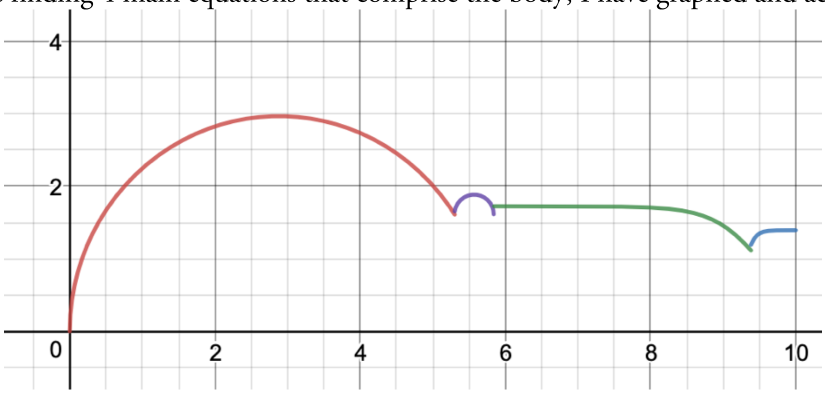
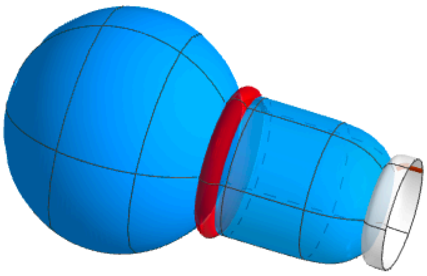


Figure 12

Unlike the graphical representation from Figure 3, above is mathematical representation. Therefore, integration and volume of revolution could be used in order to find the volume of this shape.

$$Volume\ of\ revolution = \pi \int_b^a (f(x))^2\, dx$$

 *Figure 13 – 3D model from the revolution of four equations*	3D model wascreated based on these four equations, and despite its few lacking features (face, pocket, arms), overall it looks faithful to the actual design; thus modelled well.

Volume of Revolutions

Head

$$
\begin{aligned}
V_f &= \pi \int_0^{5.303} f(x)^2 \, \mathrm{dx} \\
&= \pi \int_0^{5.303} \left(\sqrt{6.17080x - 1.08068x^2}\ \right)^2 \, \mathrm{d}x \\
&= \pi \left(\int_0^{5.303} (\, 6.17080x - 1.08068x^2) \, \mathrm{d}x \right) \\
&= \pi \left(\int_0^{5.303} 6.17080x \, \mathrm{d}x - \int_0^{5.303} 1.08068x^2 \, \mathrm{d}x \right) \\
&= \pi \left(\left[\frac{6.187080x^2}{2} \right]_0^{5.303} - \left[\frac{1.08068x^3}{3} \right]_0^{5.303} \right) \\
&= \pi(\, [3.08540x^2]_0^{5.303} - [0.36023x^3]_0^{5.303}) \\
&= \pi(\, 3.08540 \times 5.303^2 - 0.36023 \times 5.303^3) \\
&= \pi(33.04667) \\
&\approx 104
\end{aligned}
$$

Neck

$$
\begin{aligned}
V_g &= \pi \int_{5.303}^{5.840} g(x)^2 \, \mathrm{dx} \\
&= \pi \int_{5.303}^{5.840} \left(\sqrt{0.27^2 - (x - 5.5703)^2} + 1.610 \right)^2 \, \mathrm{d}x \\
&= \pi \int_{5.303}^{5.840} \left(\sqrt{0.27^2 - (x - 5.5703)^2} + 1.610 \right) \left(\sqrt{0.27^2 - (x - 5.5703)^2} + 1.610 \right) \\
&= \pi \int_{5.303}^{5.840} \left(\sqrt{0.27^2 - (x - 5.5703)^2} \right)^2 + (2 \times 1.61) \times \sqrt{0.27^2 - (x - 5.5703)^2} + 1.61^2 \, \mathrm{d}x \\
&= \pi \int_{5.303}^{5.840} (0.0729 - (x - 5.5703)^2) + 3.21\sqrt{0.27^2 - (x - 5.5703)^2} + 2.5921^2 \, \mathrm{d}x
\end{aligned}
$$

Calculation for the highlighted section:

$$= \int 3.21\sqrt{0.27^2 - (x - 5.5703)^2} \, \mathrm{d}x$$

let $u = \ 0.27^2 - (x - 5.5703)^2$

$$= 3.21\int \sqrt{u}\ \mathrm{d}x$$

$$= 3.21\int u^{\frac{1}{2}}\,\mathrm{d}x$$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -2(x - 5.5703)$$

$$\mathrm{d}x = \frac{1}{-2(x - 5.5703)}\mathrm{d}u$$

$$= 3.21\int u^{\frac{1}{2}} \times \frac{1}{-2(x - 5.5703)}\mathrm{d}u$$

$$= 3.21\left[\frac{2}{3} \times u^{\frac{3}{2}} \times \frac{1}{-2(x - 5.5703)}\right]_{5.303}^{5.840}$$

$$= 3.21\left[\frac{2}{3} \times (0.27^2 - (x - 5.5703)^2)^{\frac{3}{2}} \times \frac{1}{-2(x - 5.5703)}\right]_{5.303}^{5.840}$$

Substituting back to the original:

$$= \pi\left(\left[0.0729x - \frac{(x - 5.5703)^3}{3}\right]_{5.303}^{5.840} + 3.21 \times \left[\frac{2}{3} \times \frac{(0.27^2 - (x - 5.5703)^2)^{\frac{3}{2}}}{-2(x - 5.5703)}\right]_{5.303}^{5.840} + [2.5921x]_{5.303}^{5.840}\right)$$

$$= \pi\big((0.419196845709 - 0.392954831739) + (-0.000008173599248 - 0.000221185027877) + (15.137864 - 13.7459063)\big)$$

$$= \pi(1.786696125871635)$$

$$\approx 5.61$$

Body

$$V_h = \pi\int_{5.840}^{9.384} h(x)^2\,\mathrm{d}x$$

$$= \pi\int_{5.840}^{9.384}\left(\frac{1.72496}{1 + 0.54605e^{2.85501x - 26.79141384}}\right)^2$$

$$\approx 30.5$$

Feet

$$V_k = \pi\int_{9.384}^{10} k(x)^2\,\mathrm{d}x$$

$$= \pi\int_{9.384}^{10}\left(\frac{1.39413}{1 + (28255299675732400588 \times 10^{37})e^{-14.04145x}}\right)^2$$

$$\approx 3.63$$

Total Volume for main parts

$$V_{main} = V_f + V_g + V_h + V_k$$

$$\approx 104 + 5.6 + 30.5 + 3.63$$

$$\approx 144(3\mathrm{s.f.})$$

Calculations of Exteriors

Hand

Using the dimensions given from the tracings on *Illustrator* from Ellipse tool, the volume of the hand is below.

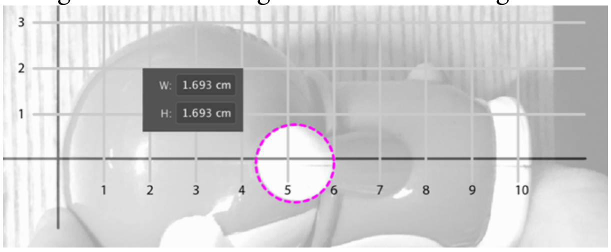


Figure 14 – The dimensions of a circle: 1.693cm x 1.693cm

$$x^2 + y^2 = \left(\frac{1.693}{2}\right)^2$$
$$m(x) = \sqrt{0.8465^2 - x^2}$$

Volume of Revolution:

$$V_m = \pi \int_0^{1.693} m(x)^2 \, \mathrm{dx}$$
$$\approx 2.54$$

Nose

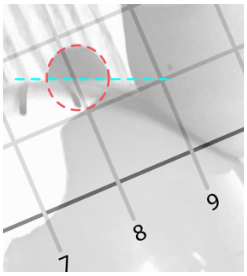


Figure 15 – Tracings of the nose

Using Ellipse tool again, the dimensions of the red circle is:

width: 0.7761 cm and height: 0.8114 cm.

The blue line goes through the center of the red circle. By setting the radius to $\frac{0.8114}{2}$, we can find out the scale factor to create a circle with same dimensions.

$$r_3 = 0.4057$$

$$\text{scale factor of } x = \frac{\frac{0.7761}{2}}{\left(\frac{0.8114}{2}\right)}$$
$$= 0.956494947$$

Figure 16 — graph of n(x)

$$y = \sqrt{(r_3)^2 - (x - r_3)^2}$$
$$n(x) = \sqrt{0.4057^2 - \left(\frac{x}{0.956494947} - 0.4057\right)^2}$$

Volume of Revolution:

Since nose is a semi-circle shape, $\frac{\pi}{2}$ is applied instead of π in volume of revolution.

$$n_v = \frac{\pi}{2} \int_0^{0.7761} \big(n(x)\big)^2$$
$$\approx 0.143$$

Total Volume for exteriors

$$V_{ex} = V_m + V_n$$
$$\approx 2.541 + 0.143$$
$$\approx 2.68 \; (3. \mathrm{s.f.})$$

Total volume (main + exteriors)

$$V_{total} = V_{main} + V_{ex}$$
$$\approx 144 + 2.68$$
$$\approx 146 (3s.f.)$$

Finding out the Volume when height is 129.3cm

Since V_{total} is a volume when height is 10cm, we can use scale factor k to find the volume when height is 129.3cm

$$k = \frac{129.3}{10}$$
$$k = 12.93$$

$$Volume\ Scale\ Factor\ (VSF) = (Linear\ Scale\ Factor)^3$$

(BBC, n.d.)

$$Volume\ when\ 129.3\text{cm} = V_{total} \times (k)^3$$
$$= 316000\ \text{cm}^3 (3s.f.)$$

Conclusion

This investigation has fulfilled the aim of finding the volume of Doraemon by utilising my strength in computer and graphic design into fields of mathematics. As a result, I was able to derive the approximate volume of Doraemon when the height is 129.3cm.

As an objective of a future industrial designer, not only appearance but also the materials that comprises the product is significant. In many ways, the industrial designer is responsible for the day-to-day interactions of customers and designed product, therefore weight would be one of the factors that can alter the design of a product. From the finding that his volume is 316000 cm^3, and the official data of weight being 129.3kg, the density is 0.4g/cm^3. Comparing with 2.7 g/cm^3 of the 2^{nd} most common used metal Aluminium.

Furthermore, it is noted officially that he has a high-tech combustion engine inside his body, thus the density of whatever metal they use will be much lighter than 0.4 g/cm^3, assuming that it is only used for encompassing. Overall, it can be concluded that even if artificial intelligence technology evolves, it is unrealistic to expect a robot like him in close future unless new strong and light alloy is developed to sustain his body elements.

Given that 1L = 1000cm^3, the amount of silicon required would be 31.6L. The volume of silicon comes in useful when casting a silicon inside the mould.

Visual Accuracy

Visual accuracy is compared as followed:

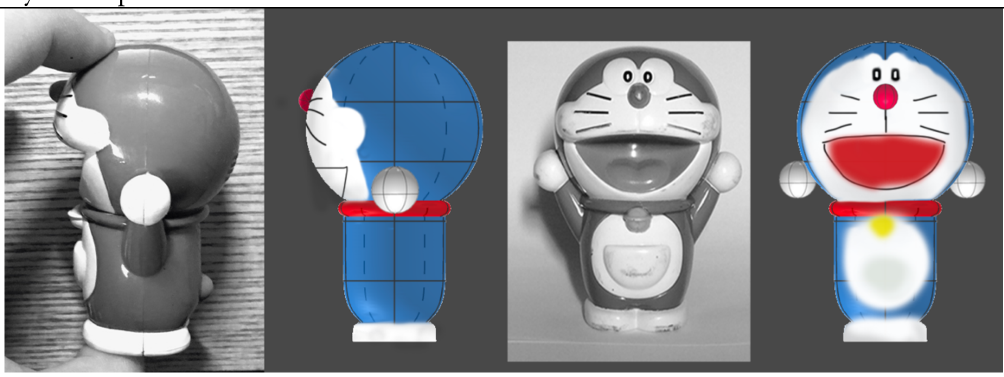


Figure 17

As seen in Figure 13, *GeoGebra* is not capable of adding details onto the 3D model it has displayed, thus I find it hard comparing the visual accuracy for it does not look like Doraemon except the shape. Therefore, in Figure 17 I coloured the model manually by at the same time maintaining the scaling. As a result, from the aesthetic point of view it is fair to say that the equations have depicted the proportions and form of Doraemon accurately when viewed from the side. This would also mark that I have successfully managed to create a 3D model out of a real-life figure or a mere sketch; which is significant for an industrial designer, because the career requires numbers of production of sketches, and modeling them on a computer before it turns into a real-life object.

Mathematical Accuracy

Although visually the model looks accurate, it remains unclear whether if it is faithful to the initial graphical model created (Figure 3). I would be able to find this through applying the method I discovered while I investigated 'Body' and 'Feet' section. Using *WebPlotDigitier,* I extracted points from the graphical model and then compared the extracted dataset with the plots the equation will give, using R^2 value. The result is following:

Table 2

BODY PARTS	R^2
Head	0.978
Neck	0.951
Body	0.995
Feet	0.965

Average R^2 for all parts
0.972

As much as it *looked* accurate, the R^2 value also show that it is also faithful to the graphical model I initially created. However, as seen in the actual 3D figure in Figure 17, although *Doraemon* does resemble circular shape, it is never a transformed circle, but a circle that is a bit inflated. While this is a source of error, it can also be seen as a strength as it fully used the current mathematics knowledge to everywhere applicable. Additionally, high R^2 value also proves that the steps I took are reliable and able to yield consistent results; which comes in significant in future purposes.

Other evaluations

Limitations	Effect	Improvement
Arm, bell, pocket was not included inside the calculation as 3D figure I own did not seem to portray the elements accurately. E.g. the side of arms were stuck to the side of the head which made it impossible to calculate the overall dimensions of his arm.	Decrease in overall volume figured out; thus, less accuracy. Small elements might seem negligible in 10cm model, but when scaled to 129.3cm it can affect in greater scale.	Use a 3D figure with arms and feet separated (less simplified model) to aim for higher accuracy.
The modelling was based on side-view of the 3D figure	I wasn't able to produce a mathematical representation which considered the width when seen from the front. As a character, there are far more many times that the character will be seen from front rather than the side.	By applying the improvement stated above, this will allow me to model from the front, as the boundaries between hand and head is clear.
The camera angle was not completely horizontal / shot from the centre of the 3D figure (The picture used in Figure 3)	Causes a warp in proportion, thus making the modeling (and the tracing) inaccurate. As camera moves off from the center, the proportions will be altered	Using a stand or anything that can stabilize the camera and figure position so that picture for modeling could be shot perpendicularly.

Moreover, this was my first time learning about logistic function. This knowledge was later applied on biology lab report when investigating the limits on photosynthesis rate, and this function supported me in justifying hypothesis.

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